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Effects of anisotropy on the formation of a lamellar phase under shear

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Abstract

We consider the phase ordering process of a system quenched into a lamellar phase in the presence of a shear flow. By studying the continuum model based on the Brazowskii free energy in a self-consistent approximation, we analyse the effects of a weak anisotropy in the quartic coupling constant, finding that it radically changes the evolution of the system.

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1. Introduction

The phase separation kinetics following a temperature quench has been longly studied for its relevance in several fields, and as a paradigm of ageing phenomena. Despite the absence of a reference theory, for binary systems the features of phase ordering are well understood. At the heart of this phenomenon is the symmetry of dynamical scaling. Specifically, the system at two subsequent times t_1 and t_2 looks statistically similar if lengths are measured in units of $R(t)$, the typical size of phase-separated regions. This symmetry is mirrored by the property of homogeneity of the structure factor, which obeys

$$C(\vec{k}, t) = t^{\alpha} f[kR(t)], \tag{1}$$

with $R(t)$ growing algebraically as $t^{1/z}$. The exponent α is related to the geometry of the growing domains of the equilibrium phase. For quenching below the critical temperature domains are compact and $\alpha = d/z$, d being the space dimensionality. In the case of complex fluids quenched into a phase with lamellar order, next to $R(t)$ another relevant length is present, the width l of the lamellae. Equation (1) then modifies to

$$C(\vec{k}, t) = t^{\alpha} f[(k - k_M)R(t)], \tag{2}$$

with $k_M \propto l^{-1}$. On the other hand, the case of phase separation in the presence of an external drive, as in the case of an applied shear flow, has only been considered quite recently. In the presence of shear, domains of the growing phases tend to align in the flow direction x breaking the rotational symmetry. Although this alters considerably the dynamics, a natural question is whether fundamental properties, such as dynamical scaling, still persist and in which form. Approximate theories [1] for binary systems suggest a natural generalization of equation (1), namely

$$C(\vec{k}, t) = At^{\alpha} f[\vec{k} \cdot \vec{R}(t)]. \tag{3}$$

Here $\vec{R}(t) = \{R_x, R_y, R_z\}$ are typical lengths in each spatial direction. One finds $R_x(t) \sim t^{1/z_{\parallel}}$ in the flow direction and $R_y(t) = R_z(t) \sim t^{1/z_{\perp}}$ in the transverse directions, with $1/z_{\parallel} = 1 + 1/z_{\perp}$, as a simple scaling analysis suggests. The property $R_y(t) = R_z(t)$ implies that rotational symmetry still holds in the plane transverse to the flow.

Ordering properties of lamellar phases in shear flow are much less understood. In this case it was argued [2, 3] that also the rotational symmetry on the plane perpendicular to the flow is broken, with lamellae aligning preferentially along a particular direction, in agreement with equilibrium expectations at high shear rates [3, 4] and simulations [5]. This issue and the possible generalization of the scaling form (2) have been considered recently [6] in the framework of the Brazowskii [7] model for an N-component order parameter, in the large-N limit. The solution, however, shows a quite unexpected behaviour. One finds that scaling is not obeyed in a three-dimensional system[4]. Typical lengths behave as $R_x(t) \sim t^{5/4}\sqrt{\ln t}$, $R_y(t) \sim \sqrt{\ln t}$, $R_z(t) = l$; this indicates the orientation of lamellae along the perpendicular direction, namely parallel to the plane formed by the flow and the shear (velocity gradient) direction, as generally found in experiments at high shear rates [9]. The appearance of logarithmic corrections and, in particular, the slow logarithmic growth of $R_y(t) \sim \sqrt{\ln t}$ are consequences of the breakdown of dynamical scaling (see section 3.1).

In this paper, we consider a model [10, 11] obtained by adding to the Brazowskii free energy an extra anisotropic coupling of strength β. This model is studied numerically in a self-consistent approximation which corresponds to the large-N limit in the case $\beta = 0$. We find that the properties with $\beta = 0$, discussed above, are radically changed by taking $\beta \neq 0$. Our data indicate that dynamical scaling is reinstated. Lamellae still orientate along the perpendicular direction but the growth laws are different from the case $\beta = 0$. One finds $R_x(t) \sim t^{5/4}$, $R_y(t) \sim t^{1/2}$, $R_z(t) = l$. We also discuss the role of thermal fluctuations and the stability of the lamellar phase. Generally, topological defects can only be stable for $N \leqslant d$. Therefore, in the large-N limit an equilibrium lamellar phase only exists at zero temperature. However, we find that the lamellar phase is stabilized for finite temperatures $T < T_c$ by the shear flow. T_c is found to depend on the kind of dynamics, namely if the order parameter is conserved or non-conserved, as already found in binary systems [12].

This paper is organized as follows: in section 2, we introduce the model with the asymmetric coupling. In section 3, after reviewing the behaviour of the Brazowskii model with $\beta = 0$, we present the results of the numerical solution of the model with $\beta \neq 0$, considering both the case with conserved order parameter (COP), which is appropriate for binary mixtures, and with non-conserved order parameter (NCOP), describing, for instance, Rayleigh–Benard cells above the convective threshold [13]. Finally, in section 4 we draw the conclusions.

[4] It must be recalled that scaling is violated also in ordinary binary mixtures (with or without shear) for systems with a conserved order parameter in the large-N limit, where a multiscaling symmetry is obeyed [8]. Here, the situation is different, since the breakdown of scaling is observed also in systems without conservation [6] of the order parameter and the multiscaling symmetry is not present.

2. The model

We consider a phenomenological model based on a coarse-grained description of the melt in terms of a concentration field $\phi(\vec{r}, t)$ playing the role of an order parameter. For an A–B diblock copolymer mixture, for instance, $\phi(\vec{r}, t)$ represents the local monomer A concentration with respect to the average. In the following, we will consider symmetric compositions with an equal concentration of the two species. We adopt a Langevin equation for the order parameter

$$\frac{\partial\phi(\vec{r}, t)}{\partial t} = -\vec{v}(\vec{r}, t) \cdot \vec{\nabla}\phi(\vec{r}, t) - \Gamma(\vec{r})\frac{\delta H[\phi]}{\delta\phi(\vec{r}, t)} + \eta(\vec{r}, t). \tag{4}$$

Here, $\vec{v}(\vec{r}, t)$ is the local velocity of the fluid. In this paper, we consider a constant plain shear flow along the x axis. Then

$$\vec{v}(\vec{r}, t) = \gamma y\hat{x}, \tag{5}$$

where γ is the shear rate and $\hat{x}$ is the unitary vector pointing along x. The Onsager coefficient $\Gamma(\vec{r})$ is assumed [14] to be a constant $\Gamma(\vec{r}) = \Gamma$ for systems with NCOP while $\Gamma(\vec{r}) = -\Gamma\nabla^2$ with COP. $\eta(\vec{r}, t)$ is a stochastic term, describing thermal fluctuations. Here, we follow the general choice of a Gaussian white noise with expectations

$$\langle\eta(\vec{r}, t)\rangle = 0 \tag{6}$$

and

$$\langle\eta(\vec{r}, t)\eta(\vec{r}\,', t')\rangle = 2T\Gamma(\vec{r})\delta(\vec{r} - \vec{r}\,')\delta(t - t'), \tag{7}$$

where T is the final temperature of the quench. This ensures that, in the absence of flow, the system attains the correct Gibbs equilibrium state. We consider the (generalized) Brazowskii Hamiltonian [7, 10, 11]

$$H[\phi] = \frac{1}{2}\int_{\vec{k}}[(r - Kk^2 + ck^4)\phi(\vec{k}, t)\phi(-\vec{k}, t)] + \frac{1}{4!}\int_{\vec{k}_1}\int_{\vec{k}_2}\int_{\vec{k}_3}\lambda(\{k_i\})\phi(\vec{k}_1, t)\phi(\vec{k}_2, t)\phi(\vec{k}_3, t)\phi(-\vec{k}_1 - \vec{k}_2 - \vec{k}_3, t), \tag{8}$$

where $\phi(\vec{k}, t)$ is the Fourier transform of $\phi(\vec{r}, t)$ and $k = |\vec{k}|$. In equation (8) $\int_{\vec{k}}$ is a shorthand for $\int_{k<\Lambda} \mathrm{d}\vec{k}/(2\pi)^d$; Λ is a phenomenological cut-off taking into account the presence of a microscopic length, such as the lattice spacing. When $K > 0$ a modulate state with wave vector $k = k_M \neq 0$ is stable. Linear analysis shows that $k_M = \sqrt{K/(2c)}$. The structure factor is defined as the second cumulant of the order parameter field

$$C(\vec{k}, t) = \langle\phi(\vec{k}, t)\phi(-\vec{k}, t)\rangle. \tag{10}$$

Inserting form (8) into equation (4), and neglecting cumulants of third and higher order [3], the equation of motion for $C(\vec{k}, t)$ reads

$$\frac{\partial C(\vec{k}, t)}{\partial t} = \gamma k_x\frac{\partial C(\vec{k}, t)}{\partial k_y} - 2\Gamma k^p\left[r - Kk^2 + ck^4 + \frac{1}{2}S(\vec{k}, t)\right]C(\vec{k}, t) + 2T\Gamma k^p, \tag{11}$$

with $p = 0, 2$ for NCOP and COP, respectively, and

$$S(\vec{k}, t) = \int_{\vec{q}}\lambda(-\vec{k}, \vec{k}, -\vec{q}, \vec{q})C(\vec{q}, t). \tag{12}$$

In order to simplify further the model, following Morse and Milner [15], we approximate $\lambda(-\vec{k}, \vec{k}, -\vec{q}, \vec{q})$ as

$$\lambda(-\vec{k}, \vec{k}, -\vec{q}, \vec{q}) \simeq \lambda[1 - \beta(\hat{q} \cdot \hat{k})^2], \tag{13}$$

where $\hat{q}, \hat{k}$ are unit vectors, and λ and β are positive phenomenological parameters that can be obtained, in principle, by microscopic calculations or by fitting to experiments. Form (12) amounts to retaining the fewest terms in a spherical harmonics decomposition of the quartic coupling constant which provides the basic physics. Anisotropy is reasonable in self-assembled systems, such as copolymers, where the interaction is non-local due to the extended nature of the polymer chains. The sign of the anisotropic term is appropriate for lamellar ordering since it results in a smaller quartic interaction for fluctuations with parallel wave vector then for perpendicular wave vectors. For $\beta = 0$, this approximation is analogous to the limit $N \to \infty$ considered in [6].

Then we have

$$S(\vec{k}, t) = \lambda \int_{\vec{q}} [1 - \beta(\hat{q} \cdot \hat{k})^2] C(\vec{q}, t). \tag{14}$$

Since the angular dependence in equation (12) is weak [10], one must have $\beta \ll 1$. Equation (10) and the self-consistency relation (13) are a closed set of equations governing the evolution of the model. From the knowledge of the structure factor several properties of the melt can be obtained. The linear dimension of equilibrated domains can be defined as

$$R_x(t) = \left[\frac{\int_{\vec{k}} k_x^2 C(\vec{k}, t)}{\int_{\vec{k}} C(\vec{k}, t)} \right]^{-\frac{1}{2}}, \tag{15}$$

and similarly for the other directions. When scaling holds, the typical length is unique, and any possible different definition gives the same result, up to constants.

The knowledge of the structure factor allows the evaluation of rheological indicators. A shear stress σ_{xy} arises due to the stretching of the domains in the direction of the flow, resulting in an increase of the viscosity $\Delta\eta = \gamma^{-1}\sigma_{xy}$ of the fluid. For a plain shear flow, the excess viscosity is given by [16]

$$\Delta\eta(t) = -\gamma^{-1} \int_{\vec{k}} k_x k_y (2ck^2 - K) C(\vec{k}, t). \tag{16}$$

3. Ordering kinetics of the lamellar phase

3.1. Overview of the behaviour with $\beta = 0$

Before discussing the behaviour of the full model it is useful to recall what is known about the case $\beta = 0$ where an explicit analytical solution at $T = 0$ is possible [6]. Since most of the behaviour of the model is analogous for NCOP and COP, we present a general discussion in the following. With $\beta = 0$, the structure factor reads

$$C^0(\vec{k}, t) = A \frac{t^2}{\ln t} \mathrm{e}^{f(\vec{k},t)+g(\vec{k},t)}, \tag{17}$$

where the superscript 0 denotes quantities computed with $\beta = 0$. Using cylindrical variables $(k_x, k_\perp, \theta)$, where $k_y = k_\perp \cos\theta$ and $k_z = k_\perp \sin\theta$, time enters the function $f(\vec{k}, t)$ only through the variables $X = k_x t^{3/2}$ and $Q = (k_\perp - k_M) t^{1/2}$, namely $f(\vec{k}, t)$ scales with respect to a parallel length $L_x^0(t) \sim t^{3/2}$ and a transverse one $L_\perp^0(t) \sim t^{1/2}$. Were $f(\vec{k}, t)$ only present, $C(\vec{k}, t)$ would obey dynamical scaling. However, this is not the case because the two functions $f(\vec{k}, t)$ and $g(\vec{k}, t)$ dominate one over the other in different time-dependent regions of k-space. Since $g(\vec{k}, t)$ does not obey the same scaling properties of $f(\vec{k}, t)$, the structure factor cannot be cast in scaling form. The behaviour of the typical lengths

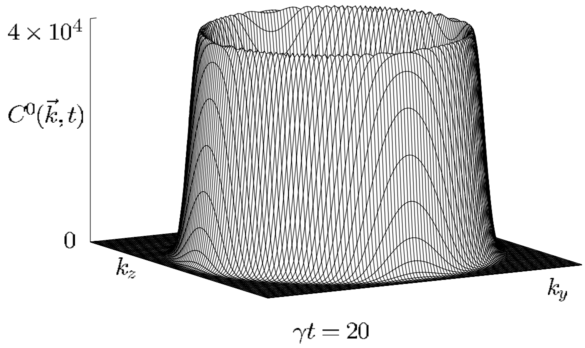

Figure 1. The structure factor $C^0(\vec{k}, t)$ of the model with NCOP and $\beta = 0$ on the plane $k_x = 0$.

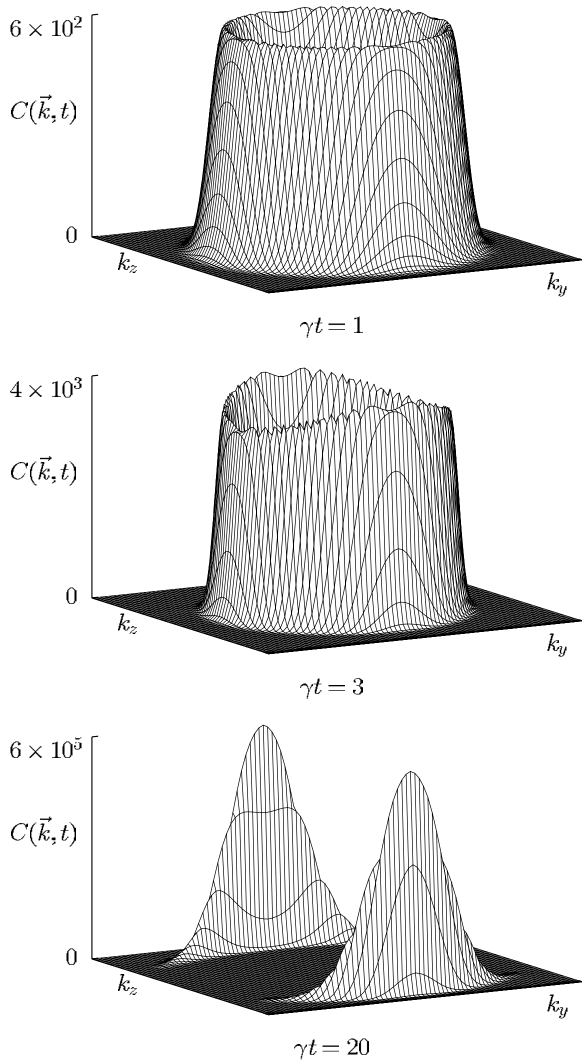

Figure 2. The structure factor $C(\vec{k}, t)$ of the model with NCOP and $\beta = 10^{-2}$ on the plane $k_x = 0$.

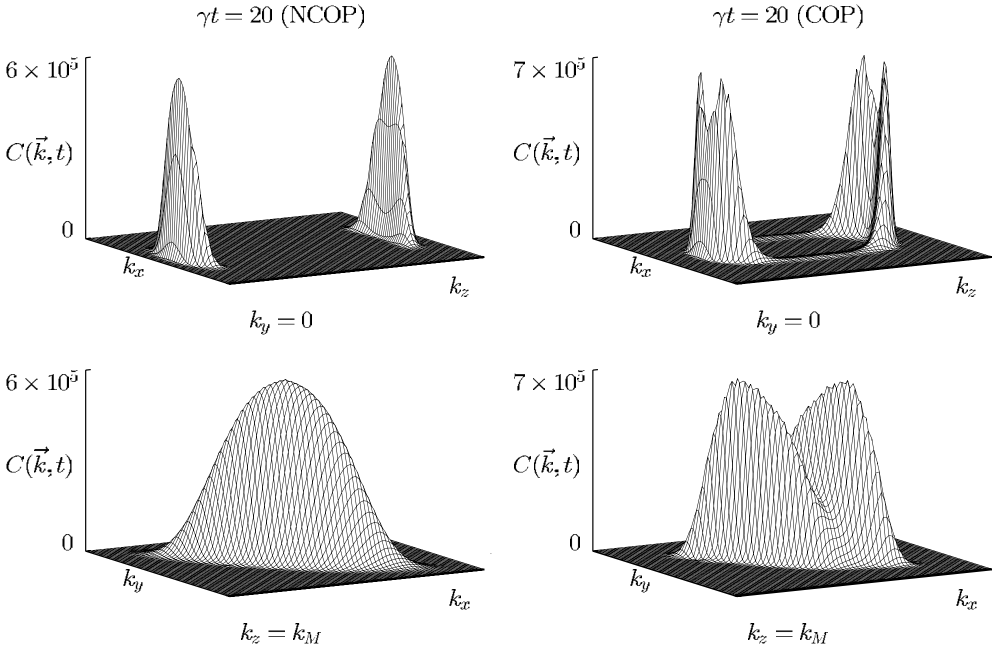

Figure 3. The structure factor $C(\vec{k}, t)$ of the model with $\beta = 10^{-2}$ on the plane $k_y = 0$ (upper figures) and $k_z = k_M$ (lower figures) for NCOP (left) and COP (right).

computed via equation (14) is found to be $R_x^0(t) \sim \gamma t^{5/4}\sqrt{\ln t}$, $R_y^0(t) \sim \sqrt{\ln t}$. The logarithmic corrections reflect the absence of dynamical scaling. In the vorticity direction $R_z^0(t)$ saturates to $1/k_M$, implying the stability of the perpendicular phase. This behaviour of the characteristic lengths, different from that of $L_x^0(t)$, $L_\perp^0(t)$, indicates that the properties of the scaling part $f(\vec{k}, t)$ alone do not determine the behaviour of the whole $C^0(\vec{k}, t)$. It is interesting to observe that, as we will see in the next section, in the case with $\beta > 0$ the growing lengths $R_x(t) \sim t^{5/4}$, $R_y \sim \sqrt{t}$ are reminiscent, apart from logarithmic corrections, of the power laws of $R_x^0(t)$ and $L_\perp^0(t)$, respectively. Regarding rheological properties, the behaviour of $\Delta\eta^0(t)$ clearly reflect the lack of dynamical scaling. In fact, assuming scaling (3), from equation (15) one finds $\Delta\eta(t) \sim [R_x(t)R_y(t)]^{-1}$. Instead, for $\beta = 0$ one finds $\Delta\eta^0(t) \sim t^{-2} \neq [R_x(t)R_y(t)]^{-1} \sim \gamma t^{5/4} \ln t$, indicating the violation of scaling. Analogously, assuming scaling (3), when the structure factor develops a maximum out of the origin, as it is usually the case for COP, its position must obey $k_x^{\max}(t) \propto R_x^{-1}$, and similarly in the other directions. Instead, in [6] it is found $k_x^{\max}(t) \sim \sqrt{\ln t/t^3}$. In the next section, we will use the behaviour of $\Delta\eta(t)$ and of the maxima of $C(\vec{k}, t)$ to infer that scaling is obeyed in the model with $\beta \neq 0$.

3.2. Numerical study of the case with $\beta \neq 0$

In this section, we consider the behaviour of the full model with $\beta \neq 0$. From the mathematical point of view, the main difference introduced by this additional term is the dependence of $S(\vec{k}, t)$ on wave vector. This makes the self-consistent closure of the model very complicated. For this reason, we resort to a numerical solution of the governing equation (10). This allows us to consider also the case of a finite temperature, which was not considered in [6].

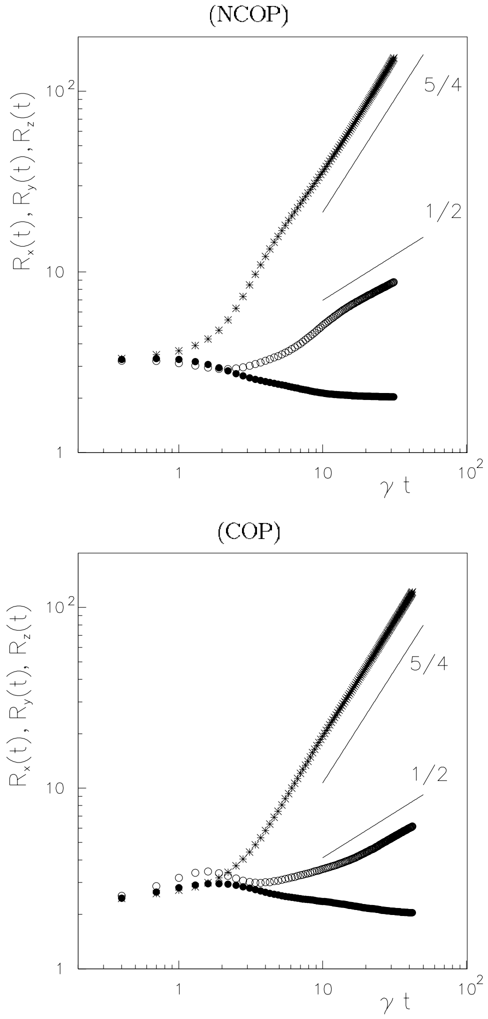

Figure 4. The characteristic lengths $R_x(t)$ (stars), $R_y(t)$ (circles), $R_z(t)$ (bullets) for NCOP (upper figure) and COP (lower figure) with $\beta = 10^{-2}$. Straight lines are power laws $t^{5/4}$ and $t^{1/2}$.

We solve the model in $d = 3$ by means of a first-order Euler algorithm. The quality of the numerical solution is very sensitive to the resolution of the grid in k space. This happens because $C(\vec{k}, t)$ grows sharp peaks in a narrow domain $D_k(t)$ which shrinks in time. The accuracy of the numerical solution, then, tends to deteriorate as time goes on because the mesh becomes inadequate to resolve $C(\vec{k}, t)$ inside $D_k(t)$. In order to improve the quality of the results, therefore, we have implemented an adapting grid technique. Fixing the number of points of the mesh, the algorithm concentrates them in the region $D_k(t)$. We do this by identifying, at each time and in each coordinate direction, the largest wave vector such that $C(\vec{k}, t)$ exceeds a certain small threshold. This wave vector, starting from the initial value Λ at $t = 0$, shrinks in time, and the accuracy of the grid improves in the wave vector sector $D_k(t)$ where $C(\vec{k}, t)$ is appreciable. Since $C(\vec{k}, t)$ decays very fast for large $\vec{k}$, one can set $C(\vec{k}, t) \equiv 0$ outside this domain. Despite the advantages of this method, solving the model is still a heavy numerical task because, at each time, the self-consistent closure

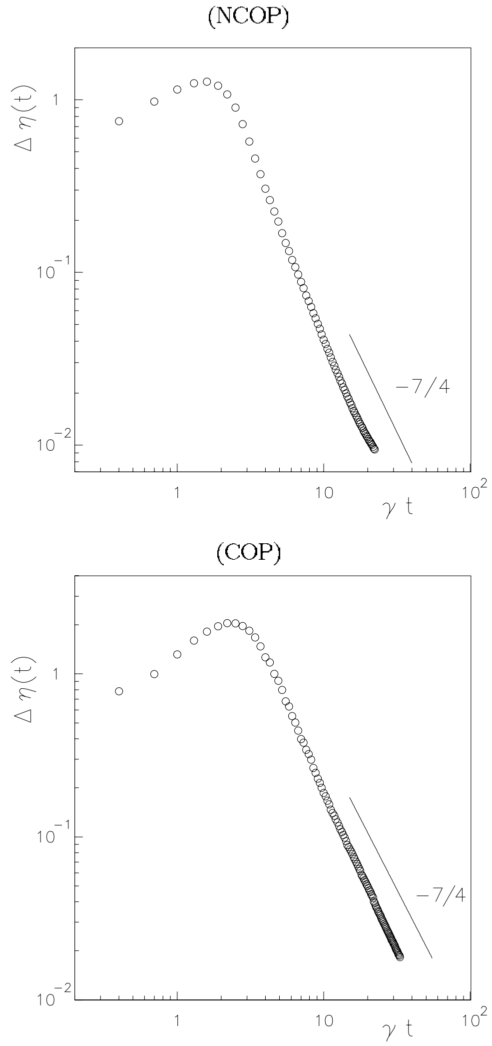

Figure 5. The excess viscosity $\Delta\eta(t)$ for NCOP (upper figure) and COP (lower figure) with $\beta = 10^{-2}$. Straight lines are the power law $t^{-7/4}$.

(13) must be enforced in order to evaluate the quantity $S(\vec{k}, t)$ to be used in equation (10). Since this implies a convolution over wave vectors, the computational time grows as the linear mesh size raised to the power $2d$, instead of the power d of the case with $\beta = 0$. We have solved equation (10) on a 73^3 points mesh. The values of the physical parameters are $\gamma = 10^{-2}, \beta = 10^{-2}$ $r = -1, c = 1, K = 0.5, \Gamma = 1$. With these parameters one has $k_M = 1/2$. We present first the results of a quench at $T = 0$.

$T = 0$. With $\beta = 0$, the equation of motion (11) is symmetric between k_y and k_z on the plane $k_x = 0$. Because of this, as shown in figure 1, on the plane $k_x = 0$, $C^0(\vec{k}, t)$ has the shape of a volcano, at each time. This symmetry is lifted in the model with $\beta > 0$. Consequently, as shown in figure 2 for NCOP, the edge of the volcano is lowered in the region $k_z \simeq 0$ until, for $\gamma t \simeq 20$, $C(\vec{k}, t)$ develops two separate peaks at $k_z = \pm k_M$, $k_y = 0$. The behaviour for COP is qualitatively similar. The features of $C(\vec{k}, t)$ on the plane $k_x = 0$ are important because

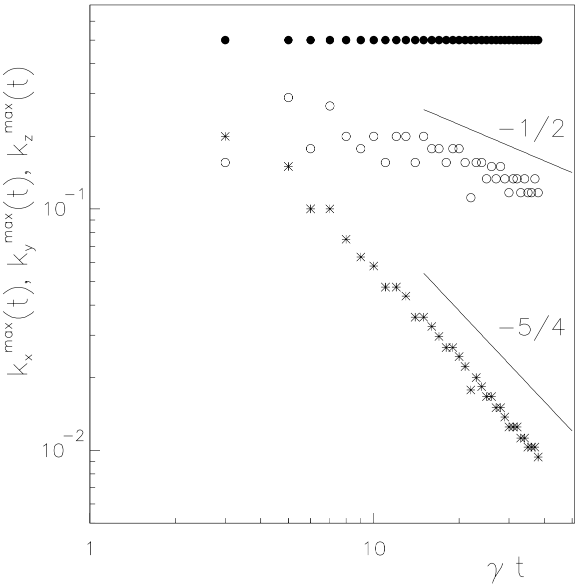

Figure 6. The locus $k_x^{\max}(t)$ (stars), $k_y^{\max}(t)$ (circles) and $k_z^{\max}(t)$ (bullets) of the maxima of the structure factor for COP with $\beta = 10^{-2}$. Straight lines are power laws $t^{-1/2}$ and $t^{-5/4}$.

this is the locus where the maxima of $C(\vec{k}, t)$ are asymptotically contained[5]. The additional symmetry of the case $\beta = 0$, therefore, is perhaps the main origin of the important differences between the two models that will be discussed below. For completeness, we report in figure 3 the evolution of $C(\vec{k}, t)$ on the planes $k_y = 0$ and $k_z = k_M$, which correspond to the other planes where the maxima of $C(\vec{k}, t)$ are found asymptotically. Here, the shape of $C(\vec{k}, t)$ is qualitatively similar to the case $\beta = 0$. With NCOP, on the plane $k_y = 0$ one observes two peaks growing at $k_x = 0, k_z = \pm k_M$, while a single peak is observed on the plane $k_z = k_M$. With COP one has the additional feature of the splitting of each peak into two, which tend to merge as time goes on. This is a consequence of the k^2 term in front of the lhs of equation (10) whose effect is to damp $C(\vec{k}, t)$ around $\vec{k} = 0$.

The behaviour of the characteristic lengths, computed through equation (14), is qualitatively similar for NCOP and COP, as shown in figure 4. After an initial transient for $\gamma t < 5$, corresponding to the linear regime where the order parameter attains local equilibrium, $R_x(t)$ and $R_y(t)$ start to increase while $R_z(t)$ saturates to k_M^{-1} from above. For large γt, we find power law behaviours compatible with $R_x(t) \sim t^{5/4}$ and $R_y(t) \sim t^{1/2}$ (best fits yield the exponents 1.28 ± 0.04 (NCOP) and 1.29 ± 0.04 (COP) for $R_x(t)$, and 0.47 ± 0.04 (NCOP) and 0.46 ± 0.04 (COP) for $R_y(t)$). The exponent of $R_x(t)$ coincides with the value found in the model with $\beta = 0$, apart from logarithmic corrections. This is also the value found in simple binary systems with COP undergoing phase separation, in the large-*N* limit. On the other hand, for $R_y(t)$ one finds a power law with a different exponent from the case $\beta = 0$. However, it must be noted that the model with $\beta = 0$ already contains a length $L_\perp(t)$ growing with the same exponent $1/2$, although, as discussed in section 3.1, this length did not determine the properties of $R_y(t)$. We also recall that the exponent $1/2$ regulates the scaling properties of the Brazowskii model without shear in the large-*N* limit.

[5] This is rigorously true only for NCOP. For COP the maxima are located in $k_x^2 = 5(1/k_M^4\gamma^2)(\ln t/t^3)$; the plane $k_x = 0$ is approached only asymptotically.

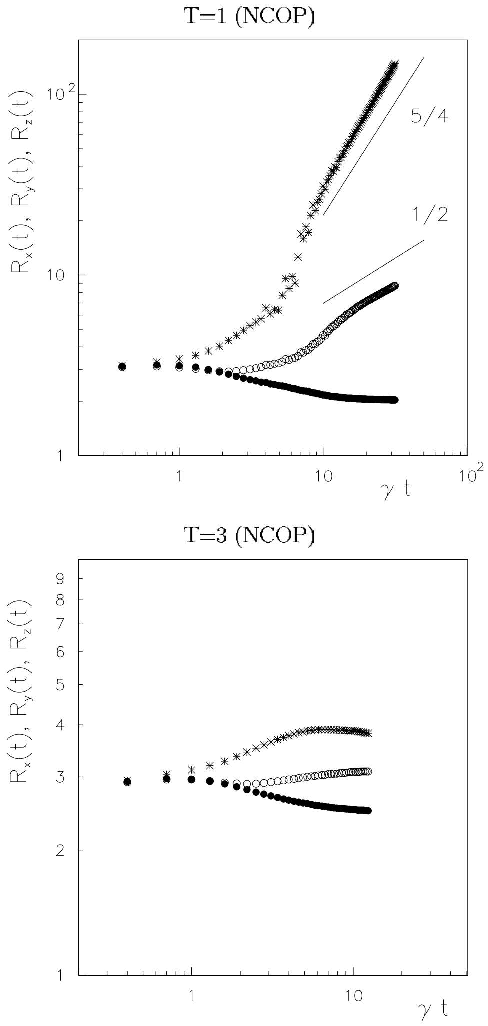

Figure 7. The characteristic lengths $R_x(t)$ (stars), $R_y(t)$ (circles), $R_z(t)$ (bullets) for a system with NCOP and $\beta = 10^{-2}$ quenched to finite temperatures $T = 1$ (upper figure) and $T = 3$ (lower figure). Straight lines are power laws $t^{1/2}$ and $t^{5/4}$.

The behaviour of $\Delta\eta(t)$, computed by means of equation (15), is shown in figure 5. According to the discussion presented in section 3.1, scaling implies $\Delta\eta(t) \propto R_x(t)R_y(t)$ asymptotically. With the power laws found for the characteristic lengths one expects $\Delta\eta(t) \propto t^{-7/4}$. Our results show that $\Delta\eta(t)$ after reaching a maximum at $\gamma t \simeq 2$ decays with a power law whose exponent, fitted from the data, is found to be 1.9 ± 0.2 for NCOP and 1.8 ± 0.2 for COP. These values suggest that dynamical scaling is obeyed.

As explained in section 3.1, another test on the validity of dynamical scaling can be made, in the case of COP, by considering the maxima of the structure factor, whose coordinates are plotted in figure 6. These data, particularly $k_y^{\max}(t)$, are noisy. By fitting them to a power law for long times we find $k_x^{\max}(t) \sim t^{-\alpha_x}$ and $k_y^{\max}(t) \sim t^{-\alpha_y}$, with $\alpha_x = 1.3 \pm 0.1$ and $\alpha_y = 0.5 \pm 0.1$. These behaviours are consistent with $k_x^{\max}(t) \sim t^{-5/4} \propto R_x(t)^{-1}$ and $k_y^{\max}(t) \sim t^{-1/2} \propto R_y(t)^{-1}$, as expected if scaling holds.

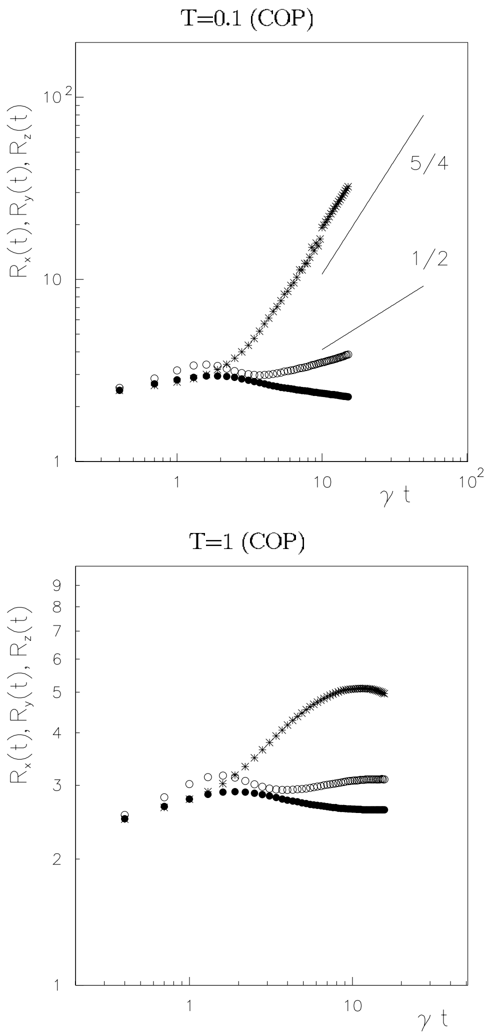

Figure 8. The characteristic lengths $R_x(t)$ (stars), $R_y(t)$ (circles), $R_z(t)$ (bullets) for a system with COP and $\beta = 10^{-2}$ quenched to finite temperatures $T = 0.1$ (upper figure) and $T = 1$ (lower figure). Straight lines are power laws $t^{1/2}$ and $t^{5/4}$.

$T > 0$. In this section, we discuss the behaviour of the model for a quench to a finite temperature $T > 0$. According to general principles [14], localized topological defects are not stable for $N > d$ in an equilibrium system. Therefore, in the absence of shear, a lamellar phase can form only at zero temperature in the large-N limit [17]. However, such general considerations do not hold for driven systems [2].

In order to study the stability of the lamellar phase under shear at finite T, we have performed a series of simulations at different temperatures. In figures 7 and 8, the behaviour of the typical lengths is plotted for different temperatures. For NCOP, at $T = 1$ the behaviour is very similar to the case $T = 0$: the typical lengths grow with the same power laws. This suggests that the lamellar phase orders with the same modalities of the case $T = 0$. The same conclusion is obtained by the comparison of $C(\vec{k}, t)$ or $\Delta\eta(t)$ in the two cases. On the other hand, when the temperature is raised up to $T = 3$ a qualitatively different behaviour is observed. Here, $R_x(t)R_y(t)$ and $R_z(t)$ saturate to a constant value for large times. This implies that the lamellar structure is not growing, and the system attains a stationary disordered state. Therefore, one can argue that the critical temperature T_c for the stability of the lamellar phase is finite. For COP the situation is qualitatively similar, the only quantitative difference being the value of T_c which is lower than for NCOP. In fact, the typical lengths saturate already at $T = 1$. The dependence of T_c on dynamics must not be worrisome, since the system approaches a stationary state that is not in equilibrium, due to the presence of the flow. An analogous dependence is found in binary systems [12].

4. Conclusions

In this paper, we have considered the ordering kinetics of a system quenched into a lamellar phase by studying numerically a model based on the Brazowskii free energy with the addition of an anisotropic coupling constant of strength β. The model is analysed in a self-consistent approximation which formally linearizes the theory. For $\beta = 0$, this approximation is equivalent to the large-N limit for an N-component vector order parameter which was studied analytically in a previous paper [6]. We find that the presence of the anisotropic term radically changes the behaviour of the model. This basic difference is probably mainly due to the lifting of the symmetry between k_y and k_z on the plane $k_x = 0$. Our result suggest that dynamical scaling, which was violated for $\beta = 0$, is recovered. The lamellar phase orders along the perpendicular orientation, as for $\beta = 0$, but the growth laws of the characteristic lengths of the lamellae are different. We find $R_x(t) \sim t^{5/4}$ and $R_y(t) \sim t^{1/2}$, while $R_z(t)$ saturates to the typical width of lamellae. The power growth law of $R_y(t)$, in particular, is very different from the logarithmic behaviour found for $\beta = 0$. The excess viscosity, after reaching a maximum, decays algebraically as $[R_x(t)R_y(t)]^{-1} \sim t^{-7/4}$. These results apply to systems with NCOP and COP as well, showing that the presence of the conservation law is almost irrelevant for the ordering of a lamellar phase; this extends what was already known [17] in the case without flow to sheared systems. We have also considered the role of temperature fluctuations. For quenches to finite temperatures the system orders similarly to the case $T = 0$ up to a characteristic temperature T_c which is found to be different for NCOP and COP. The dependence of T_c on the dynamics was already shown in the case of binary systems [12]; here the same phenomenon is found for the ordering kinetics of a lamellar phase.

As far as the ordering properties of the lamellar phase are considered, the presence of a weak anisotropy in the Brazowskii free energy gives rise to a different and articulated pattern of behaviours with respect to the isotropic case. This suggests that the same model in different conditions, as in the microemulsion phase, may behave in a different and novel way. It would be interesting to study the effects of anisotropy on the properties of these phases.

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In some experiments on surfactant lamellar systems a double orientation transition (parallel/perpendicular/parallel) has also been observed increasing the shear rate, see Berghausen J, Zipfel J and Linder P and Richtering W 1998 *Europhys. Lett.* **48** 683 In our studies, we do not find changes of orientation during the evolution
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